Statement of Purpose

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I am applying for admission to the PhD studentship in Mathematics under the research group of Prof Chris Wendl at Humboldt University, because I intend to pursue a research career in mathematics, broadly geometry and topology. Specifically, I am interested in low dimensional topology, symplectic topology and the theories of Floer homology.

In my MSc thesis, I am working under Prof Sushmita Venugopalan towards understanding in detail the theory of pseudo holomorphic curves and their properties, from McDuff and Salamon's *J-holomorphic curves and symplectic topology* (both the big and small books). Since its introduction in 1985 by Mikhail Gromov, this theory has been of great use in symplectic topology, most notably in the form of Gromov's nonsqueezing theorem. I am currently studying about the moduli space of J-holomorphic curves, and in the process, developing the necessary analysis background for myself, most importantly some Fredholm theory and techniques in PDEs. I intend to study linearly till Gromov compactness in the book, and then focus on Floer (co)homology. I had come across Floer theory and its applications in symplectic topology from a mini course offered by Prof Alexandru Oancea at CMI in my first year of Masters study and now intend to understand this in detail.

The introduction of pseudoholomorphic curves has led to theories of symplectic invariants in many contexts. It was one of the guiding factors for Floer's proof of the Arnold conjecture, a very classical problem in the theory of Hamiltonian systems. The conjecture relates periodic orbits of a time dependent Hamiltonian DE to the cohomology for a symplectic manifold. J-holormophic curves have also found applications in theory of invariants for 4-manifolds and Gromov-Witten theory in mathematical physics.

The above interest was partly kindled by my summer study of complex/Kahler geometry under Prof Sushmita, from Voisin's *Hodge Theory and Complex Algebraic Geometry I.* I studied the first part of the book, leading upto Hodge theory, and my study has continued because of my current ongoing course on complex geometry with Prof T R Ramadas. As a project in this course, I intend to develop my knowledge of elliptic partial differential operators and study in detail a proof of the Calabi conjecture, following Dominic Joyce's *Compact manifolds with special holonomy.*

I have always been interested in geometry and topology in the broadest sense, and have tried to utilize opportunities to learn various perspectives in the field. In my second semester of MSc, I got interested in the topology of lower dimensional manifolds, and read the first few chapters of Dale Rolfsen's *Knots and links.* I also studied some theory of mapping class groups for surfaces and the Dehn-Lickorish theorem which says that surface homeomorphisms are generated by Dehn twists. I got interested in classification problems, in various (topological, smooth) contexts, and started reading Milnor's h-cobordism theorem and Hatcher's notes on 3-manifold topology. I was always fascinated with Morse theory (I studied it first from Milnor's book in my BSc), a very simple to understand idea with profound consequences. Following Hatcher's notes, I gave a talk outlining the prime decomposition of 3-manifolds and the necessary Morse theory at the *Young Topologists' Meet* at CMI.

My graduate studies have been built upon and motivated by a series of basic courses I have taken in my BSc and first year of MSc. In my second semester of MSc, I took a course on sheaf theory with Prof Ramadas, which developed the theory of derived categories, hypercohomology and reformulations of classical treatments of cohomology in this language. Towards the end, I did a directed reading of intersection (co)homology for stratified spaces, focusing on Deligne's treatment using sheaves, following the book *Intersection cohomology*, by Borel et al.

At the beginning of my MSc, I got inculcated into a classical algebraic perspective of theories. I attended a solid graduate course in algebra under Prof Shiva Shankar, which helped make homological considerations second nature for me. At the same time, I started attending a very interactive category theory seminar under Prof Upendra Kulkarni, following Emily Riehl's *Category Theory in Context*. His way of looking at categorical objects along with hindsight of past courses made me realise the perspective of looking at different cohomology theories as sort of projectoins of a "universal" cohomology theory (the Grothendieck "motives").

My BSc studies have been in a variety of disciplines, in and out of topology. I always tried to challenge myself with the hardest courses I could take, and over time, have gained the maturity and comfort with large subjects like algebraic topology. I studied, in my third semester of BSc, homology and cohomology from *Hatcher*, till Poincare duality. This continued, in my fifth semester of BSc, when I took a topics course in algebraic topology under Prof Shiva Shankar, where we focused on hands-on computations, the proofs of Poincare duality and other forms of duality (like Alexander duality), connections of singular cohomology with differential geometry (following the book *Differential Forms in Algebraic Topology*). Finally, intending to show equivalence of singular and de Rham cohomology for appropriate spaces, we introduced sheaves and studied them in a basic sense to the point of proving the above using fine resolutions, and covered Cech cohomology for sheaves.

In my final semester of BSc, I took an introductory course on symplectic geometry with Prof Sushmita Venugopalan, studying from Cannas Da Silva's *Lectures on symplectic geometry*. I was interested in the physics flavor taken in the course, and am pursuing this interest in my current studies. For two months, I visited ENS Paris after my BSc, under the CMI-ENS program which selects the top 3 students of the graduating batch. Under Prof Ilia Itenberg of ENS, I studied the topology of real algebraic varieties, closely following Oleg Viro's online notes on the same. As a further exploration of the combinatorial aspects of the subject, I studied the first few chapters of Itenberg's *Tropical Algebraic Geometry*.

I have been deeply interested in mathematics since high school, and have had great opportunities to learn various topics from leading researchers. Every year for the last four years has opened new directions for me in this vast subject.

In my PhD, I would like to work on low dimensional topology using the tools of symplectic topology and analysis, though I am open to working on related areas of geometry. With the necessary training, I hope to make significant contributions to the areas I work in, and share knowledge with fellow researchers and students.